Double Diffusive Phenomenon in a Gravity Modulated Environment

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Abstract: Analytical investigation of onset of double diffusive convection in a two component two phase system under gravity modulation to study the effects of salinity gradient and temperature gradient ; the gradients are of opposite nature employing the method of normal mode and the modified perturbation technique is presented. Analysis is carried out for Viscous , Brinkmann and Darcy models by deriving solvability condition and computing the first non – zero correction to the Rayleigh number. Possibility of enhancing or suppressing convection by suitable choice of the governing parameters is studied.

Indexterms : Boussinesq fluid saturated porous layer, double-diffussive convection, gravity modulation, modulation parameter,salinity parameter, stress-free boundaries,diffusivityratio,porous parameter,

INTRODUCTION

This chapter deals with the analytical investigation of the double-diffusive phenomenon in a gravity-modulated environment. The continuum model incorporates all the necessary characteristic features / properties of the medium. The system is a two-component, two-phase system in a modulated environment. The two gradients of salinity and temperature present in the system are of opposing nature

We employ modified perturbation technique and carry out analysis for Viscous, Brinkmann and Darcy models. Its known that Double diffusive convection exhibits several interesting features in an unmodulated environment. The solvability condition is derived and the first non-zero correction to the Rayleigh number computed.

MATHEMATICAL FORMULATION:

The physical configuration consists of a Boussinesq fluid saturated porous layer of infinite horizontal extent, subject to destabilizing temperature and stabilizing concentration gradients. The layer is confined between two plates situated at z=0 and z=d respectievely. Further, the layer is under the influence of periodically varying gravitational field. The boundaries are assumed to be stress-free.

Under suitable assumptions and approximations, the governing equations of motion are :

The conservation of momentum

$$\rho_0\left(\frac{D\vec{q}}{Dt}\right) = -\nabla P + \rho\vec{g} + A_1\mu\nabla^2\vec{q} - A_2\frac{\mu}{k}\vec{q} \qquad (1)$$

The conservation of energy

$$\frac{\partial T}{\partial t} + (\vec{q}\,.\nabla)T = \kappa\nabla^2 T \qquad (2)$$

The conservation of solute

$$\frac{\partial C}{\partial t} + (\vec{q}\,.\nabla)C = \kappa_s\nabla^2 C \quad (3)$$

The equation of continuity

$$\nabla\,.\,\vec{q} = 0$$

$$and \qquad (4)$$

The equation of state

$$\rho = \rho_0\,[1-\alpha\,(T-T_0) + \alpha_s(C-C_0)] \qquad (5)$$

Where

$$\vec{g} = g_0\,[1+\epsilon\,\cos\omega^* t\,]\,\hat{k} \quad (6)$$

Nomenclature: (The symbols have the following meaning)

(x , y , z), the space variables ;

$\vec{q} = (u, v, w)$, the velocity of the fluid or the mean filter velocity of the fluid layer (in case of a porous layer) ;

$\vec{g}$, *the* gravitational acceleration ;

g_0, *the*constant part of gravity;

t , the time ;

C , the specific heat ;

$\hat{k} - the\ unit\ vector\ in\ the\ z - direction$;

k -the permeability of the medium ;

κ − the effective thermal conductivity of the fluid in the presence of porous medium

T ,$T_b\theta$ - the temperature ;

C ,C^* - the concentration ;

ρ, ρ_0 -the density and the mean density of the fluid ;

μ -the dynamic viscosity ;

$\nu = \frac{\mu}{\rho_0}$ - the kinematic viscosity ;

ε^*, ω^*, t^* - the dimensional amplitude , frequency and time ;

d -the layer thickness

κ, κ_s -the thermal and solutal diffusivities

$P = \frac{\nu}{\kappa}$ the Prandtl number

$R^* = \frac{\alpha g_0 (\Delta T) d^3}{\nu\kappa}$ − the thermal Rayleigh number

$R_s = \frac{\alpha_S g_0 (\Delta C) d^3}{\kappa\nu}$ the solute Rayleigh number

$\tau = \frac{\kappa_s}{\kappa}$ –the diffusivity ratio

$\sigma^2 = \frac{d^2}{k}$ – the porous parameter

The basic state solution is$\bar{q} = (0,0,0), T = T_b(z)$ and C = $C_b(z)$where in there is a balance between the buoyancy force and the pressure.

Introducing a small perturbation on the basic state so that

$\bar{q} = \bar{q},\ T = T_b + \theta,\quad C = C_b + C^*\quad \rho = \rho_b + \rho^1\ and\ P = P_b + P^1$ (7) Where $\bar{q}$, θ, C^*, ρ^1 andP^1 represent small deviations from static state due to convective motion. Pressure is eliminated from the momentum equation and the resulting equations after non-dimensionalisation through an appropriate scale we have

$$\nabla . \bar{q} = 0 \qquad (8)$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = \omega \qquad (9)$$

$$\left(\frac{\partial}{\partial t} - \tau\nabla^2\right) C = \omega \qquad (10)$$

$$\left(P^{-1} \frac{\partial}{\partial t} - A_1\nabla^2 + A_2\sigma^2\right)\nabla^2\omega = (1 + \epsilon \cos \omega t)(R\nabla_1^2 - R_S\nabla^2\theta) \qquad (11)$$

Eliminating ω& c from the above equations we have

$$\left(P^{-1} \frac{\partial}{\partial t} - A_1\nabla^2 + A_2\sigma^2\right) \left(\frac{\partial}{\partial t} - \nabla^2\right) \left(\frac{\partial}{\partial t} - \tau\nabla^2\right)\nabla^2\theta = (1 + \epsilon \cos \omega t) \left[R\nabla_1^2 \left(\frac{\partial}{\partial t} - \tau\nabla^2\right)\theta - R_S\nabla_1^2 \left(\frac{\partial}{\partial t} - \nabla^2\right) \theta\right] \qquad (12)$$

which's an eighth order differential equation in θ with the bouudary conditions in dimensionless form on velocity , temperature and concentration are :

$$\theta = C = 0 \text{at } z = 0, 1 ; \omega = 0 = \frac{d^2\omega}{dz^2} \qquad (13a,b)$$

or

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$\theta = \frac{d^2\theta}{dz^2} = \frac{d^4\theta}{dz^4} = \frac{d^6\theta}{dz^6} = 0$(14)

STABILITY ANALYSIS :

Linear stability analysis of double-diffusive porous convection is performed under a modulated environment using an asymptotic procedure.The critical conditions for the three models of viscous, Brinkman and darcy are computed.Mathematical and physical validity of the solutions are discussed.

In view of $\in < 1$, we consider the asymptotic expansion in powers of $\in$, for variables R and θ as

$$(R, , C) = (R_0, \theta_0, C_0) + (R_1, \theta_1, C_1) + \in^2 (R_2, \theta_2, C_2) + \ldots\ldots (15)$$

Substituting (15) into (12) we obtain the following set of differential equations corresponding to) $\in^0$, $\in$ and $\in^2$:

$L\,\theta_0 = 0$ (16)

$L\theta_1 = [(fR_0 + R_1)(\frac{\partial}{\partial t} - \tau\nabla^2) - R_S(\frac{\partial}{\partial t} - \nabla^2) f]\nabla_1^2\theta$ (17)

$L\theta_2 = \nabla_1^2[(\frac{\partial}{\partial t} - \tau\nabla^2)\{(fR_0+R_1)\theta_1 + (fR_1+R_2)\theta_0\} -- R_S(\frac{\partial}{\partial t} - \nabla^2) f\theta_1]$(18)

Where

$L = (\frac{\partial}{\partial t} - A_1\nabla^2 + \sigma^2 A_2)\nabla^2 (\frac{\partial}{\partial t} - \tau\nabla^2)(\frac{\partial}{\partial t} - \nabla^2) - R_0\nabla^2(\frac{\partial}{\partial t} - \tau\nabla^2) + R_S\nabla_1^2(\frac{\partial}{\partial t} - \nabla^2)$(19)

Applying normal mode analysis, solution of ( ) is obtained. On letting $\theta_0^n = \sin n\pi z$ in (19), we get on simplification

$L = A_1(n^2\pi^2 + a^2) + \sigma^2 A_2\}((n^2\pi^2 + a^2)^3 + R_S(n^2\pi^2 + a^2) = R_0 a^2\,\tau(n^2\pi^2 + a^2)$(20)

Or

$R_0 = (A_1\frac{(n^2\pi^2+a^2)^3}{a^2}) + (A_2\sigma^2\frac{(n^2\pi^2+a^2)^2}{a^2} + \frac{R_s}{\tau})$(21)

The least eigen value for a fixed wave number occurs at n =1.

Now, $\frac{dR_0}{da^2} = 0 \rightarrow a_c = \pi$ and $R_{oc} = 4\pi^2\sigma^2$(25a,b)

We now discuss the following three cases :

Case (1) Fluid Layer :$A_1 = 1, A_2 = 0$.

From (21) we obtain $R_0 = \frac{(n^2\pi^2+a^2)^3}{a^2} + \frac{R_s}{\tau}$(22)

In the absence of concentration gradient (i.e. $R_S = 0$) above equation exactly coincides with that of the classical Benard problem.

Now, $\frac{dR_0}{da^2} = 0 \rightarrow a_c = \frac{\pi}{\sqrt{2}}$ and $R_{oc} = \frac{27\pi^4}{a^2} + \frac{R_s}{\tau}$ (23a, b)

Case (2) Darcy model :$A_1 = 0, , A_2 = 1$

For a densely packed two-component fluid saturated porous layer, we obtain from ( ) :

. $R_0 = \sigma^2\frac{(\pi^2+a^2)^2}{a^2} + \frac{R_s}{\tau}$(24)

Case (3) :Brinkmann model :: $A_1 = 1 = A_2$

For a sparsely packed double diffusive porous layer, we obtain from (1.1.22) :

$$R_0 = (\pi^2 + a^2 + \sigma^2)\frac{(\pi^2 + a^2)^2}{a^2} + \frac{R_s}{\tau} (26)$$

Again, $\frac{dR_0}{da^2} = 0 \rightarrow a_c^2 = \frac{1}{4}[(9\pi^4 + 10\sigma^2\pi^2 + \sigma^4)^{\frac{1}{2}} - (\pi^2 + \sigma^2)]$

Convection is possible only when$(9\pi^4 + 10\sigma^2\pi^2 +$ *σ412>π2+σ 22(27)*

This condition always holds since$(\pi^2 + \sigma^2) > 0$ (28)

HIGHER ORDER SOLUTIONS :

We now consider the first and second order system of differential equations are considered and the solutions are computed.

From (17) $L\theta_1^{(n)} = -a^2[(fR_0 + R_1)\tau(n^2\pi^2 + a^2) - R_s f(n^2\pi^2 + a^2)]\theta_0^{(n)}$(29)

R_1 ,R_3 vanish due to orthogonality condition.We can write

$L(\omega, n) = P^{-1}\omega^2[(1+\tau)(n^2\pi^2 + a^2)^2] + [\{A_1(n^2\pi^2 + a^2) + A_2\sigma^2\}\{\omega^2 - \tau)(n^2\pi^2 + a^2)^2\}(n^2\pi^2 + a^2) + \tau(n^2\pi^2 + a^2)\{A_1(\pi^2 + a^2)^3 + A_2\sigma^2(\pi^2 + a^2)^2 + \frac{R_s}{\tau}a^2\} - R_s a^2(n^2\pi^2 + a^2)] + i\,\omega\,[(n^2\pi^2 + a^2)\{-P^{-1}\omega^2 + \tau P^{-1}(n^2\pi^2 + a^2)^2\} + (1+\tau)(n^2\pi^2 + a^2)^2(A_1(n^2\pi^2 + a^2) + A_2\sigma^2) - \{A_1(n^2\pi^2 + a^2)^3 + \sigma^2 A_2)(n^2\pi^2 + a^2)^2 + \frac{R_s}{\tau}a^2\} + R_s a^2]$ (30)

Thus $L(\omega,n) = \omega^2 X_n + Y_n + i\,\omega Z_n^{\frac{1}{2}}$ where $X_n = (1+)\,P^{-1}N_1^2$(31a,b)

$Y_n = A_1\tau\,(N_2^3 - N_1^3)N_1 + A_2\sigma^2\tau\,(N_2^2 - N_1^2)N_1 + \omega^2 N_1 N_3$(31c)

$Z_n^{\frac{1}{2}} = [N_1^2 N_3 + A_1(\tau N_1^3 - N_2^3) + A_2\sigma^2(\tau N_1^2 - N_2^2) + R_s a^2 \frac{\tau-1}{\tau} + P^{-1}N_1(\tau N_1^2 - \omega^2)]$(31d)

$N_1 = (n^2\pi^2 + a^2), N_2 = (\pi^2 + a^2), N_3 = (A_1 N_1 + A_2\sigma^2)$ (31e, f, g)

Letting- $R_s \to 0,\ \tau \to 0$, we find that

$X_n = P^{-1}N_1^2,\quad Y_n = A_1 + A_2\sigma^2(N_2^2 - N_1^2)N_1 + \omega^2 N_1 N_3$, (32a,b)

$Z_n^{\frac{1}{2}} = [N_1^2 N_3 - A_1 N_2^3 - N_2^2 + A_2\sigma^2 - + P^{-1}N_1\omega^2$.(32c)

Further it follows that $L(\sin n\pi z e^{-i\omega t}) = L(\omega,n)(\sin n\pi z e^{-i\omega t})$ (33)

From (29), (31), (33), we obtain

$\theta_1^{(n)} = -a^2(n^2\pi^2 + a^2)(R_0\tau - R_s)\,Re[\sum \frac{e^{-i\omega t})(\sin n\pi z}{L(\omega,n)}]$(34)

$L\theta_2^{(n)} = -a^2(n^2\pi^2 + a^2)\{\tau(f R_0\theta_1 + R_2\theta_0) - R_s f(n^2\pi^2 + a^2)\theta_1\}$(35)

We derive the solvability condition by applying the condition that the RHS of (34) is orthogonal to $\sin\pi z$.Thus

$R_2\tau(n^2\pi^2 + a^2)\int_0^1 \sin^2\pi z\, dz = (n^2\pi^2 + a^2)(R_0\tau - R_s)\int_0^1 \overline{f\theta_1}\sin\pi z\, dz$ (36)

Or

$R_2 = -2(R_0 - \frac{R_s}{\tau})\int_0^1 \overline{f\theta_1}\sin\pi z\, dz$ (37)

Let's compute $\overline{\theta_1 L\theta_1} = -\ a^2(R_0\tau - R_s)(n^2\pi^2 +$ a2fθ1sinπz (38)

$\overline{f\theta_1}\sin\pi z = \frac{\overline{\theta_1 L\theta_1}}{(R_0\tau - R_s)\tau(n^2\pi^2 + a^2)}$ (39)

But from (34), we have

$\overline{\theta_1 L\theta_1} = (R_0\tau - R_s)^2 a^4 Re\,[\,(n^2\pi^2 + a^2)[\sum \frac{e^{-i\omega t}\sin n\,\pi z}{L(\omega,n)}$] (40)

Hence we have

$R_2 = \frac{2(R_0\tau - R_s)^2(n^2\pi^2 + a^2)}{\tau}\sum \frac{\cos^2\omega t\,(\omega^2 X_n + Y_n)}{(\omega^2 X_n + Y_n)^2 + \omega^2 Z_n}\int_0^1 \sin^2\pi z\, dz$ (41)

Or $R_2 = \frac{(R_0\tau - R_s)^2(n^2\pi^2 + a^2)}{2\tau}\sum \frac{(\omega^2 X_n + Y_n)}{(\omega^2 X_n + Y_n)^2 + \omega^2 Z_n}$ (42)

Results are presented through graphs.

RESULTS AND DISCUSSION :

In this section, the results are discussed. In figures 1.1 to 1.11, the graphs of $\frac{R_2}{R_0}$versusω is presented for different values of the salinity gradient, diffusivity ratio,,porous parameter and the Prandl number. The results correspond to the fluid/viscous and Brinkman models. The effect of frequency modulation is studied in the range $0< \omega < 100$ *The results predict the following.*

(1) The graphs present in figures 1.1 to 1.3 correspond to the viscous /fluid model.From figure 1.1 it is clear that instability is more in the range $20< \omega <40$whereas for other values of ω, the motion is more of supercritical nature and the maximum positive correction to R_0 occurs for $\omega \approx 0$ which corresponds to the unmodulated case.Here R_s=10 and $\tau = 0.1$.

(2) As $R_s$takes the values 100 and 1000, the behavioural pattern also changes drastically.For $R_s = 10^3$ and $\tau = 0.1$, the system becomes highly unstable (to subcritical motions) for all values of P in the range $0< \omega <20$

(3) From figures 1.4 and 1.5 it is apparent that for small Prandtl numbers, the salinity gradient has a remarkable influence on the behavioural pattern. There is a drastic difference in the values of the ratio for R_s =10, 100.It is observed that for very small values of the frequency modulation parameter the ratio takes large values.In other words, the first order correction to the Rayleigh number is sufficiently small in the case of gravity-modulated environmentwhen compared to values in the unmodulated environment.Therefore, by the suitable choice of governing parameters it is possible to enhance or suppress convection.

(1) In figures 1.6 to 1.11 the graphs are drawn for $\sigma = 10, 25, 40;$ *and* for $R_s =$

$10, 10^3$.One important thing observed in these figures is, the graph has the same profile in the range 20≤ $\omega \leq$ 40, whatever may be the combination of parameters. It is found that for large values of $\sigma \geq (25)$all the curves merge and the effect of Prandtl number becomes insignificant.

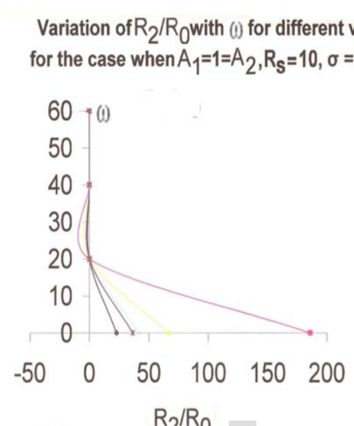


FIG 5.1

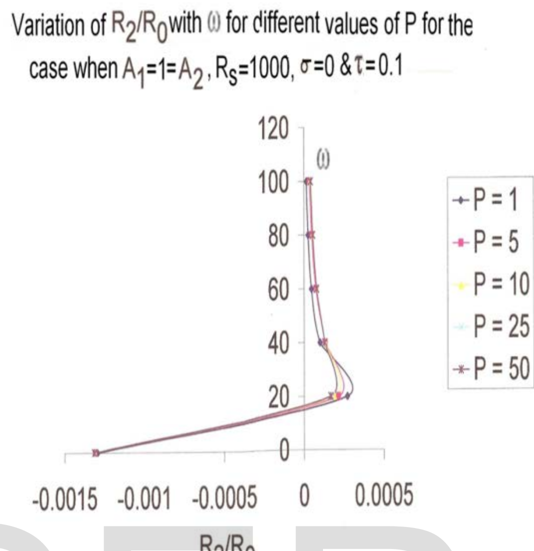


FIG 5.3

Variation of R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=100$, $\sigma = 0$ & $\tau = 0.1$

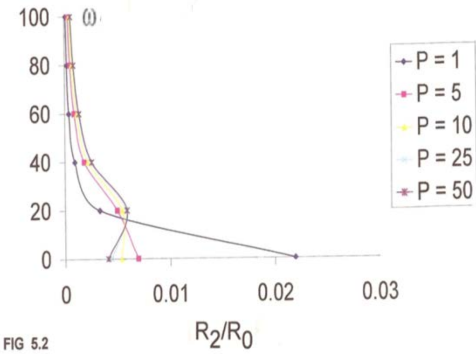


FIG 5.2

Variation of R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=10$, $\sigma = 5$ & $\tau = 0.1$

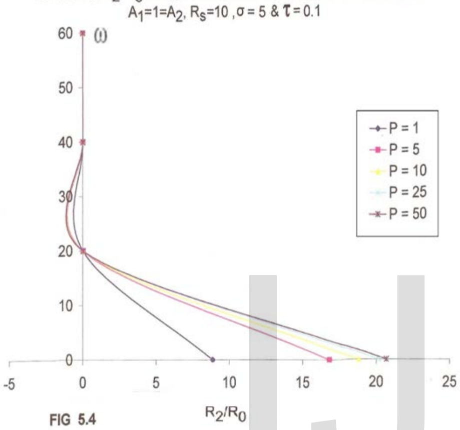


FIG 5.4

Variation of R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=10$, $\sigma = 10$ & $\tau = 0.1$

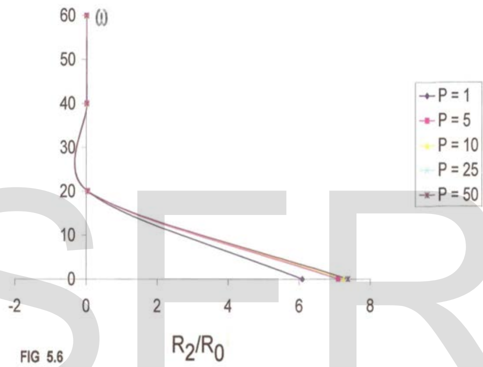


FIG 5.6

Variation with R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=100$, $\sigma = 5$ & $\tau = 0.1$

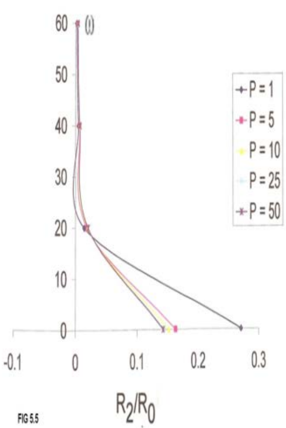


FIG 5.5

Variation of R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=100$, $\sigma = 10$ & $\tau = 0.1$

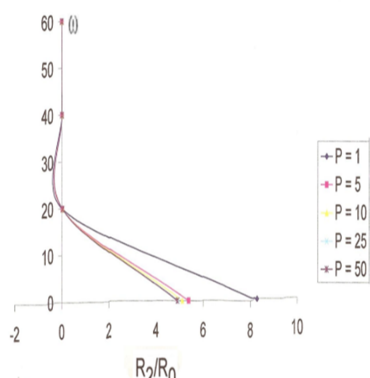


FIG 5.7

Variation of R_2/R_0 with ω for different values of P for the case when $A_1=1=A_2$, $R_s=100$, $\sigma = 25$ & $\tau = 0.1$

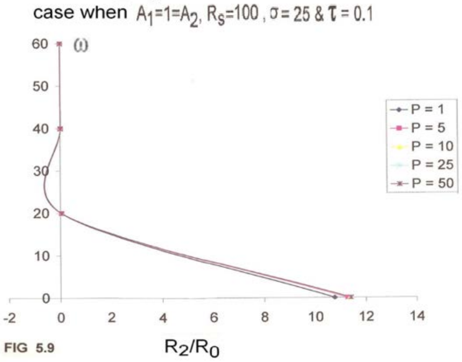
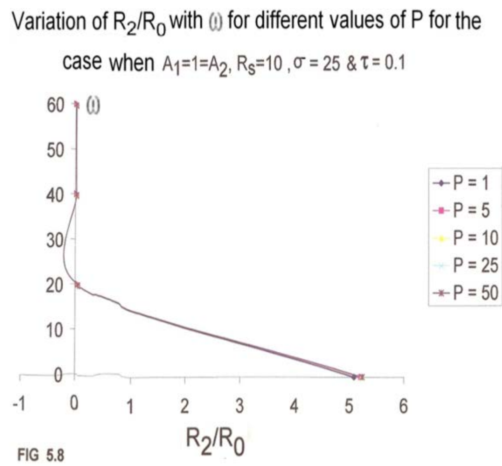
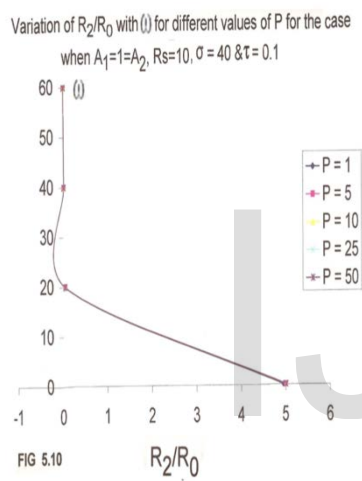


FIG 5.9





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